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EXAMPLE OF ANTIPHASE FLUCTUATION — NEGATIVE ELECTRIC CURRENT IN DIELECTRIC

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The possibility of the fluctuational appearance of an order parameter which is unusual for a given system is illustrated by the example when the electric current arising in an irradiated dielectric is directed opposite to the applied electric field.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Пример антифазной флуктуации — отрицательный электрический ток в диэлектрике

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Возможность флуктуационного появления параметра порядка, необычного для данной системы, проиллюстрирована примером, когда электрический ток, возникающий в облученном диэлектрике, направлен против приложенного электрического поля.

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The phase state of a system can be characterized by the quantity called the order parameter. When the sign of the latter becomes opposite to that one which is typical of the state given, one says about antiphase fluctuations. This term has been used, for instance, by Cook /1/ considering pretransitional clusters associated with structural phase transitions. Such fluctuations are generally local in space as well as in time. Although in some oversimplified cases they may be local only in space, as it takes place /2/ for several exact solutions to the time-dependent Landau-Ginzburg model of phase transitions. The exact solutions mentioned represent solitons moving in one-dimensional space. In other cases antiphase fluctuations do not move but are fixed in occasionally chosen regions of a system. This is so for systems having special stochastic properties, when one comes across the problem of Chinese postman, i.e., achievement of the minimal energy of a postman delivering the mail along a set of streets connecting cross-roads. Then the energetical advantage leads to the formation of a random phase-antiphase state as for some spin-glass models /3/, where spin-up domains coexist with spin-down and spin-zero ones.

Time-dependent antiphase fluctuations occurring in a stationary regime can be taken into account by the quasi-equilibrium theory of heterophase fluctuations /4/. And strongly nonstationary fluctuations should be described by nonequilibrium dynamical equations. Here a nonstationary case will be considered: the case of irradiated dielectric in an external electric field.

Irradiation can lead to the transition of a dielectric to a weak conductor. Another interesting result is the appearance under irradiation of regions of disorder /5-9/, so that the matter becomes the heterophase mixture of two phases with different densities.

The first situation is investigated here, when only electric properties are important. As an order parameter putted into the correspondence to the dielectric-conductor transition the conductivity or the electric current could be chosen. In order to illustrate the main idea — the possibility of a strange behaviour of the induced electric current — let us use the simple model /10/ of the dielectric with injected charges.

The case under consideration is described by the electrodynamic equations

$$\operatorname{div}\vec{D} = 4\pi\rho, \quad \operatorname{div}\vec{j} + \frac{\partial\rho}{\partial t} = 0, \tag{1}$$

in which $\vec{D}$ is the electric induction, ρ is the density of injected charges and $\vec{j}$ is the density of their electric current. The material equations are as follows:

$$\vec{D} = \epsilon\vec{E}, \quad \vec{j} = \mu\rho\vec{E}, \tag{2}$$

where ϵ is the dielectric constant, μ is the mobility constant or simply the mobility. Let the dielectric be bounded by two plates perpendicular to the x-axis and at the distance L from each other. The potential difference be unchanged:

$$\int_0^L E_x(x,t)\,dx = V_0 > 0. \tag{3}$$

The injected density $\rho(\vec{r},t) = \rho(x,t)$ is initially situated in the form of the thin layer at $x = x_0$ with the surface charge density σ_s:

$$\rho(x,0) = \sigma_s\delta(x - x_0). \tag{4}$$

As the macroscopic order parameter the density of the total electric current

$$J_{tot}(t) = \int_0^L j_x(x,t)\,dx \tag{5}$$

is taken. In order to simplify the notation the dimensionless quantities are introduced:

$$\xi \equiv \frac{x}{L}, \quad a \equiv \frac{x_0}{L}, \quad \tau \equiv \frac{\mu V_0 t}{L^2},$$

$$E \equiv \frac{E_x L}{V}, \quad n \equiv \frac{4\pi L^2 \rho}{\epsilon V_0}, \quad \sigma \equiv \frac{4\pi L \sigma_s}{\epsilon V_0}, \quad J \equiv \frac{4\pi L^2 J_{tot}}{\mu \epsilon V_0}.$$

Then (1) and (2) give

$$\frac{\partial E}{\partial \xi} = n, \quad \frac{\partial}{\partial \xi}(nE) + \frac{\partial n}{\partial \tau} = 0. \tag{6}$$

The boundary condition (3) is now

$$\int_0^1 E(\xi, \tau)\, d\xi = 1, \tag{7}$$

and the initial condition (4) is

$$n(\xi, 0) = \sigma\, \delta(\xi - a). \tag{8}$$

The order parameter (5) transforms to

$$J(\tau) = \int_0^1 n(\xi, \tau) E(\xi, \tau)\, d\xi. \tag{9}$$

In the place of (9) one might write

$$J = nE + \frac{\partial E}{\partial \tau}. \tag{10}$$

This is because $\partial J / \partial \xi = 0$, and therefore, $J \equiv J(\tau)$.

The set of equations (6) and (10) with boundary and initial conditions (7) and (8), respectively, can be solved using the method of characteristics. The solution shows that the charged layer widens in time, the charge density diminishes according to the law $n \sim 1/\tau$. The positions of the left and right surfaces of the layer move in such a way that at the moment τ_1 the left surface touches zero: $\xi_\ell(\tau_1) = 0$; at the moment τ_2 it comes off the plane $x = 0$, so that again $\xi_\ell(\tau_2) = 0$; at the moment τ_3 the right surface touches the right boundary $x = L$, therefore $\xi_r(\tau_3) = 1$; finally, at τ_4 the left surface reaches the right plane: $\xi_\ell(\tau_4) = 1$, and the electric current vanishes. Thus, $0 < \tau_1 < \tau_2 < \tau_3 < \tau_4$.

The expressions for the total current (9) or (10) inside the corresponding time intervals have the following forms. In the first interval

$$J = \sigma(1 + \sigma a - \frac{\sigma}{2})\, e^{\sigma\tau} \qquad (0 \leq \tau \leq \tau_1),$$

the time τ_1 being defined by the equation

$$\sigma^2 \tau_1 = \sigma - 2 + (2 + 2\sigma a - \sigma)\, e^{\sigma \tau_1}.$$

In the second interval

$$J = -2\frac{d^2}{d\tau^2}\left(\frac{\tau}{u}\ \frac{du}{d\tau}\right) \qquad (\tau_1 \le \tau \le \tau_2),$$

where $u = J_0(\sqrt{2\tau}) + AY_0(\sqrt{2\tau})$, $J_0(\cdot)$ and $Y_0(\cdot)$ are the Bessel functions of the first and second order, respectively, the constant A should be found from the equation

$$\left(2\frac{du}{d\tau} + \sigma u\right)_{\tau_2} = 0,$$

and the moment τ_1 is given by the relation

$$\tau_2 = \left[u^2\frac{d^2u}{d\tau^2} \Big/ \left(\frac{du}{d\tau}\right)^2\right]_{\tau_2}.$$

In the third interval

$$J = \sigma_1\left(1 + \frac{\sigma_1^2\tau_2}{2} - \frac{\sigma_1}{2}\right)e^{\sigma_1(\tau-\tau_2)} \quad (\tau_2 \le \tau \le \tau_3),$$

here

$$\sigma_1 = \sigma - \left[\frac{2}{u}\ \frac{du}{d\tau}\right]_{\tau_1}^{\tau_2},$$

the time τ_3 must be determined from the equation

$$\sigma_1^2\tau_3 = \sigma_1 + 2 + (\sigma_1 - 2 - \sigma_1^2\tau_2)e^{\sigma_1(\tau_3-\tau_2)}.$$

And the fourth interval·

$$J = 2\frac{d^2}{d\tau^2}\left(\frac{\tau}{v}\ \frac{dv}{d\tau}\right) \qquad (\tau_3 \le \tau \le \tau_4),$$

where $v = J_0(\sqrt{2\tau}) + BY_0(\sqrt{2\tau})$, the constant B is defined by the equation

$$\left(2\frac{dv}{d\tau} + \sigma_1 v\right)_{\tau_3} = 0,$$

the final time τ_4 satisfies the condition $(dv/d\tau)_{\tau_4} = 0$.

As is evident, the electric current can become negative, that is directed opposite to the external field, in the first time interval and in the part of the second interval when either

$$\sigma > \frac{2}{1-2a},\ a < \frac{1}{2}, \text{ or } -\frac{2}{2a-1} < \sigma < 0,\ a > \frac{1}{2} (J = 0, \quad \sigma = \frac{2}{1-2a}, a < \frac{1}{2}).$$

The physical reason for the appearance of the negative electric current is that the injected charges themselves form the electric field that forces them to move in the direction opposite to the applied field.

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